The dissemination of Louis Bachelier's work in economics and mathematics.

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Louis Bachelier is doubtless the best known French mathematician in the history of modern finance theory. His work is known for the application to stock exchange operations that he proposed as early as 1900 in his doctoral thesis.

The canonical history of financial economics – the history created from a canon of texts and developed at the time financial economics – casts Bachelier as a formidable forerunner who was forgotten until the mid-1950s. His "rediscovery" is attributed to the American mathematician Leonard J. ("Jimmie") Savage who, on coming across Bachelier's work published in 1914, sent a postcard to his economist colleagues. However, Bachelier's contribution to the development of scientific ideas has still not been accurately assessed. The main reason for this is that the dissemination of Bachelier's work has not been clearly established. This is precisely my purpose: to examine the dissemination of Bachelier's work in order to better assess his impact on the development of science.

Dissemination of Bachelier's work since 1900

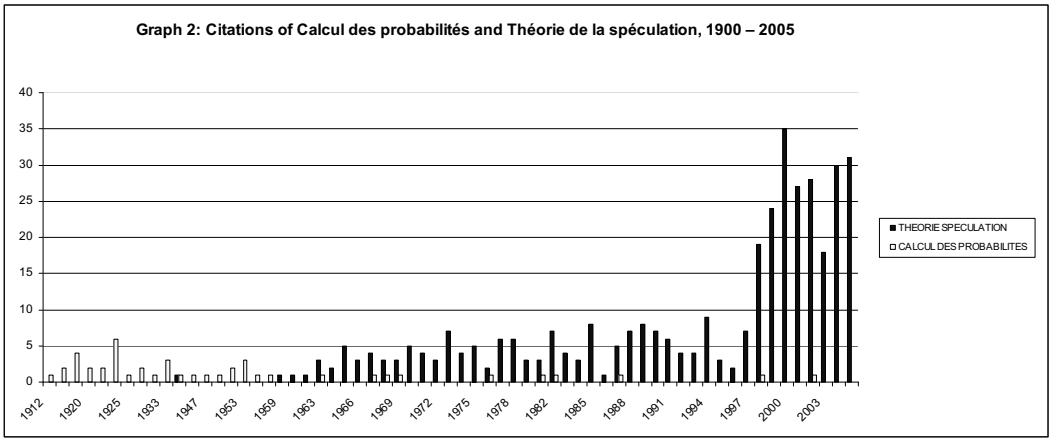
Contrary to the received notion given credence by the canonical history of financial economics, Bachelier's work has never been forgotten; on the contrary, as the following graph shows, dissemination of his work began in 1912, the year his *Calcul des probabilités* was published, and has not ceased since. This graph also allows us to distinguish four periods in the use of Bachelier's work.

- The first period (1912 – 1923) is marked by a growing dissemination of Bachelier's work.
- The second period (1924 – 1960) exhibits a discontinuous and relatively weak dissemination of Bachelier's work.
- The third period (1961 – 1997) is marked by a renewed interest in Bachelier's work, cited without interruption and more frequently. The highlight of this period is the publication in 1964 of Paul Cootner's *The Random Character of Stock Market Prices,* in which Bachelier's thesis was translated into English for the first time.
- The fourth and final period (1998 – 2005) is marked by continuous referencing and an explosion in the number of citations of Bachelier's publications.

Moreover, this graph shows that Bachelier was first known for *Calcul des probabilités*, and that his thesis began to be cited only in 1959, after which point *Calcul des probabilités* was barely cited at all – except for a single citation in 1937. Looking only at Bachelier's two main publications, then, two very distinct periods in the dissemination of his work can be discerned:

- 1912 to 1959, when only *Calcul des probabilités* was cited;
- 1959 onwards, when the thesis has been almost the sole publication cited.

These two periods coincide with the four periods, because the break at the end of the 1950s is apparent here also. Let us now look more closely at this break.

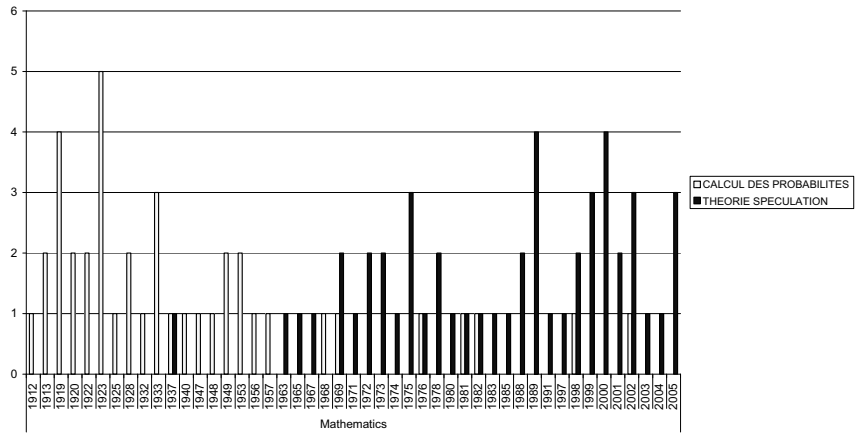


The influence of Bachelier's work

Bachelier and mathematics

Three elements characterize the dissemination of Bachelier's work in mathematics journals (graph 5).

Graph 5: Citations of Calcul des probabilités and Théorie de la spéculation in mathematics journals, 1900 – 2005



- First element: in the first part, we saw that Bachelier's works began to be cited from 1912 onwards. This early use of Bachelier's work occurred in articles published in mathematics journals.

- Second element: we observe that *Calcul des probabilités* achieved relatively good dissemination among mathematicians.

- Third element: we observe that, with one exception, articles published in mathematics journals until 1957 cited *Calcul des probabilités* exclusively, totally ignoring Bachelier's thesis; and then, from 1983 onwards, with only a few exceptions, Bachelier's thesis alone was cited.

This periodization (prior to 1987 and after 1963) corresponds with the division into four periods identified. In order to understand this periodization, we need to bear in mind Bachelier's scientific aim on the one hand and the context in which he published his works on the other.

Let us begin by examining the scientific aim of Bachelier's work. He studied mathematics and mathematical physics and defended his thesis in mathematical physics. Considering his education, he was therefore a mathematician, not an economist or financial analyst. Similarly, his research program dealt with mathematics alone: his aim was to construct a general, unified theory of the calculation of probabilities exclusively on the basis of continuous time.

The sequence of events in his work on his scientific aim was as follows.

His first publication, *Théorie de la spéculation*, which was also his doctoral thesis, introduced continuous time probabilities by demonstrating the equivalence between the results obtained in discrete time and in continuous time.

His 1901 article, "Théorie mathématique du jeu," enabled him to generalize the first results contained in his thesis by moving systematically from discrete time to continuous time and by adopting what he called a "hyperasymptotic" point of view.

In 1912, Bachelier published *Calcul des probabilités*. It was through this book that mathematicians learned of Bachelier's work. The object of *Calcul des probabilités* was to "make known new methods and new results that represent […] a complete transformation of [the calculation of probabilities]. The basis of these new studies is the conception of continuous probabilities […]".

This review of Bachelier's scientific aim underlines the mathematical character of Bachelier's research. Although his mathematics research program undoubtedly originated from his interest in financial markets, it is clear that, once embarked upon his scientific career, Bachelier took no further interest in financial markets other than for the mathematical results he deduced from them. To understand the mathematical importance of his works, let us now look at the context in which they were published.

As mentioned earlier, Bachelier was interested in the calculation of probabilities in continuous time. However, development of continuous time probability theory did not truly begin until 1931. Bachelier was thus a forerunner. Yet he was not the only mathematician seeking to reconstruct traditional probability theory: a number of publications aimed at renewing this theory emerged between 1900 and 1930. During this period, several authors were working on random variables and on the generalization of the central limit theorem. Louis Bachelier was one of the firsts to propose continuous time results, on Brownian motion in particular. However, up until the 1920s, his work remained known and accessible only to a few specialists. The 1920s, then, were a period of very intensive research into probability theory – and into continuous time

probabilities in particular – that paved the way for the construction of modern probability theory.

Modern probability theory was properly created in the 1930s, in particular through the work of Kolmogorov, who proposed its main founding concepts. In constructing some of his new concepts Kolmogorov used Bachelier's work, which he considered to be the first study of stochastic processes in continuous time, and which he generalized in his 1931 article. From these beginnings in the 1930s, modern probability theory developed and became increasingly influential. But it was not until after World War II that the Kolmogorov's axioms became the dominant paradigm in this discipline. It is also after World War II that the American probability school was born in the United States. It was led by Doob and by Feller, both of whom cited Bachelier's work very early on. These two writers had a major influence on the construction of modern probability theory, particularly through their two man books published in the early 1950s which proved, on the basis of the framework laid down by Kolmogorov, all results obtained prior to the 1950s, thereby enabling them to be accepted and integrated into the discipline's theoretical corpus.

This context reveals the full importance and originality of Bachelier's scientific aim regarding the mathematical theory of probability calculation. In 1900, when Bachelier was defending his thesis, continuous time probabilities were in their infancy and probability theory as a whole was a discipline undergoing total reconstruction. Between the end of the 19th century and the 1930s, the only work being carried out in this new field was the particularly innovative work of mathematicians and physicists. Bachelier was one of these mathematicians, as was his thesis supervisor, Henri Poincaré.

The first thing to note is that throughout the period in which modern probability theory emerged and developed – from the turn of the 20th century through to the 1930s – *Calcul des probabilités,* the sole publication of Bachelier to be cited, was used by mathematicians. In other words, *Calcul des probabilités* formed part of the reference works for mathematicians at the time this new discipline was being constructed.

Another significant element is that Bachelier's works were cited by the period's main contributors to modern probability theory and are often associated with some of the greatest probability theorists of the time, underlining the fact that Bachelier's work was considered sufficiently important and innovative by mathematicians at the time. Subsequently, from the end of the 1950s, citations of *Calcul des probabilités* ceased, with Bachelier's thesis alone being cited from then on. This singular break raises the following question: what explanation can there be for the fact that writers suddenly ceased using the work of an author who had served as a reference and from then on only cited his first published work, even though no references had been made to this work in the past?

We have seen that Bachelier's work was disseminated through the *Calcul des probabilités*. I have also explained that during the 1940s and 1950s, mathematicians rigorously proved the main results obtained by Bachelier, thereby making modern probability theory more accessible. The manner in which a work can be used is very informative in this regard: by citing Bachelier's doctoral thesis, writers highlighted the priority of Bachelier's work, that is, the fact that he was the first to propose results that subsequently became common knowledge (Brownian motion, continuous time probabilities, etc.). Writers then therefore looked for the first publication by Bachelier to

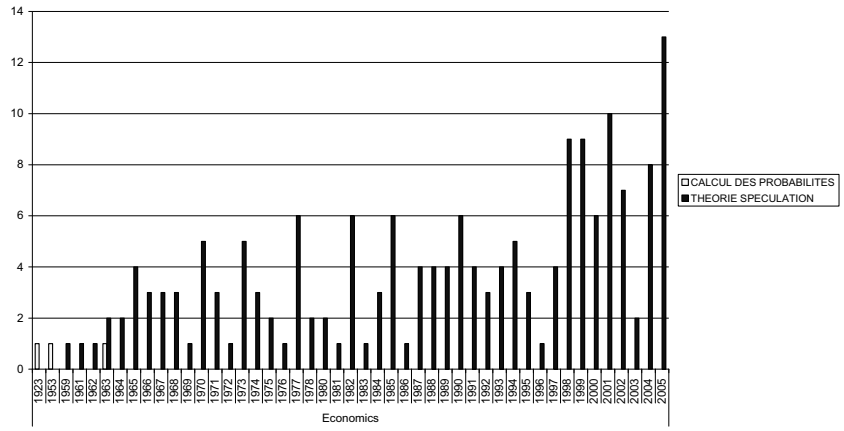
deal with continuous time probabilities, independently of this first publication's influence; this explains why Bachelier's thesis only began to be cited when Bachelier's mathematical work was no longer influencing research work in this field.

We may conclude, then, that Bachelier's work was initially used by mathematicians because it was at the leading edge in its field and thus constituted a vital reference (which explains why Bachelier's name is mentioned along with those of other great mathematicians). Later, once modern probability theory had been sufficiently developed, mathematicians drew on this new work and no longer on Bachelier's work, which explains why his *Calcul des probabilités* ceased being cited.

Bachelier and economics

Generally speaking, throughout the entire period, articles published in economics journals cite almost exclusively Bachelier's thesis. Furthermore, graph 8 shows that economists only began to cite Bachelier's work from the 1960s onwards, with the exception of two instances, one in 1923 and the other in 1953 – which, moreover, cite *Calcul des probabilités* and not *Théorie de la spéculation*. Lastly, it is only from 1961 onwards that Bachelier's works are cited in economics journals without discontinuity.

Graph 8: Citations of Calcul des probabilités and Théorie de la spéculation in economics journals, 1900 – 2005



Two questions arise with regard to the dissemination of Bachelier's work among economists. First, what explains this belated interest in Bachelier's work by economists? Second, knowing that Jimmie Savage, a mathematician at Chicago University, is considered responsible for the discovery of Bachelier's work by economists in the 1960s, what impact did Savage have in economists' discovery of Bachelier?

I have already shown that it cannot be asserted that Bachelier's work had remained unknown, since *Calcul des probabilités* was cited from 1912. And yet, one might assume that, because citations of Bachelier's thesis did not appear until the late 1950s, the applications of Bachelier's work to financial markets were unknown. Again, this is not the

case, since *Calcul des probabilités* re-presents all the results contained in the thesis. Also, the absence of citations of Bachelier's thesis does not imply ignorance of the possible applications of his work to financial markets. Moreover, it was mathematicians, such as Savage, who drew the attention of economists to this application of the developments of probability theory. However, contrary to the belief generally held since the canonical history was developed and circulated, Savage was not the first to have brought the usefulness of Bachelier's work for the study of financial markets to the attention of economists: Bachelier's work was applied to analyze financial markets as early as the 1920s.

In December 1922 a session on mathematical statistics was held at the seventh annual meeting of the Mathematical Association of America. Arne Fisher presented a mathematical formula introduced by Bachelier. This means that the absence of references by economists to Bachelier's work prior to the 1950s cannot be explained by ignorance of its possible application to financial markets. The problem lies elsewhere, and must be sought by looking at the development of modern probability theory.

Economists were unable to read the new mathematics developed in Bachelier's doctoral thesis until the 1960s. Consequently, the application of continuous time probabilities to financial markets could not be performed by economists. As mentioned earlier, it was only from the 1950s onwards that nonspecialists, and hence economists, began using the tools of modern probability theory. Knowing this gives us a better picture of Jimmie Savage's "rediscovery" of Bachelier in the mid-1950s. Since Bachelier was already known to American mathematicians, it is reasonable to assume that Savage, as a mathematician, had been familiar with Bachelier's mathematical work for some time. Why, then, did he send his famous postcard to bring Bachelier to the attention of his economist colleagues? Almost certainly because at the time the potential applications of Bachelier's work to financial markets were unknown to virtually all economists, and few mathematicians had drawn attention to this potential. Savage sent his postcard at a time when some mathematicians were beginning to apply the new mathematics developed in the first half of the 20th century to social sciences. Savage was one of their number and it was his research in mathematics (and more specifically his research into the application of mathematics to social sciences) that led him to look at the application of Bachelier's work to stock market operations. Savage therefore played a role in disseminating Bachelier's work from one discipline to another. It is not surprising, then, that Bachelier's work in finance should be "discovered" by economists from the late 1950s, nor that this discovery came via a mathematician, for whom a reading of Bachelier's work was more accessible.

However, as we saw earlier, at the time when economists began using stochastic processes and modern probability theory, Bachelier's *Calcul des probabilités* was no longer being referred to by mathematicians, who were now citing only Bachelier's thesis. *Calcul des probabilités* was no longer cited because Bachelier's results either had been superseded, or had been rewritten in language that integrated Kolmogorov's axiomatic system of probability calculation and subsequent developments. Therefore, people were no longer reading Bachelier, but other mathematicians. A perfect illustration of this point is the case of the mathematician M.F.M. Osborne, who in 1959 published his article on Brownian motion in the stock market; he was unaware of Bachelier's work but referred to more recent results. Furthermore, when the application of Bachelier's work to finance was rediscovered, his mathematical work had lost its innovative character; *Théorie de la spéculation* was at this point cited to provide historical perspective.